Heat Transfer Calculator

Theory and Equations

Ozgur Tumuklu
Rensselaer HAVA Laboratory

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Abstract

This document explains the physics behind the three tabs of the Heat Transfer calculator on the HAVA Laboratory website: *Rayleigh flow* (ratio tables for compressible flow with heat addition), *Rayleigh flow with heat addition* (exit state and choking heat for a given inlet state), and *Pipe convection* (Nusselt number, heat transfer coefficient and the temperature change along a heated pipe). For each, it states the assumptions, derives the equations the calculator evaluates, describes how the flow behaves, and gives the limits of validity.

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1 Rayleigh flow: heat addition in compressible duct flow

Rayleigh flow is the steady, one-dimensional flow of a gas through a duct of constant cross-section in which heat is added or removed, with no wall friction and no work. It is the standard model for combustors in ramjets and scramjets, afterburners and gas heaters, where the dominant effect on the flow is the change of stagnation temperature.

1.1 Assumptions

- Steady, one-dimensional flow in a constant-area duct; properties are uniform over each cross-section.
- No wall friction (friction without heat transfer is Fanno flow, in the Frictional Flows calculator), no shaft work, no body forces.
- Calorically perfect gas: $p = \rho RT$ with constant c_p, c_v and $\gamma = c_p/c_v$. Heat addition does not change the gas composition.
- Heat q is added per unit mass between station 1 (inlet) and station 2 (exit); $q > 0$ is heating, $q < 0$ cooling.

1.2 Governing equations

Between any two stations, conservation of mass, momentum and energy give

$$\rho_1 u_1 = \rho_2 u_2, \qquad p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2, \qquad h_1 + \tfrac{1}{2}u_1^2 + q = h_2 + \tfrac{1}{2}u_2^2. \tag{1}$$

With $h = c_p T$, the energy equation states that the heat added changes only the stagnation temperature:

$$q = c_p\,(T_{02} - T_{01}), \qquad T_0 = T\left(1 + \frac{\gamma - 1}{2}M^2\right). \tag{2}$$

Since $a^2 = \gamma RT$, the momentum flux is $\rho u^2 = \gamma p M^2$, and the momentum equation becomes

$$p\,(1 + \gamma M^2) = \text{constant}. \tag{3}$$

Together with the mass equation and the ideal-gas law, every property can then be written as a function of the Mach number alone.

1.3 Ratios to the sonic state

Every state is compared with the state the same flow would reach at $M = 1$, marked with a star. It is the state reached by adding exactly enough heat to make the flow sonic; it need not exist in the actual duct. In terms of γ and M only:

$$\frac{p}{p^*} = \frac{1+\gamma}{1+\gamma M^2}, \qquad \frac{T}{T^*} = M^2\left(\frac{1+\gamma}{1+\gamma M^2}\right)^2, \qquad \frac{\rho^*}{\rho} = \frac{u}{u^*} = \frac{(1+\gamma)M^2}{1+\gamma M^2}, \tag{4}$$

$$\frac{T_0}{T_0^*} = \frac{2(1+\gamma)M^2\left(1 + \frac{\gamma-1}{2}M^2\right)}{(1+\gamma M^2)^2}, \qquad \frac{p_0}{p_0^*} = \frac{1+\gamma}{1+\gamma M^2}\left(\frac{2 + (\gamma-1)M^2}{\gamma+1}\right)^{\gamma/(\gamma-1)}. \tag{5}$$

The entropy difference from the sonic state follows from $s - s^* = c_p \ln(T/T^*) - R \ln(p/p^*)$:

$$\frac{s^* - s}{R} = \ln \frac{p}{p^*} - \frac{\gamma}{\gamma - 1} \ln \frac{T}{T^*} \;\geq\; 0. \tag{6}$$

Because T_0^* and p_0^* are fixed for a given flow, the change between two stations is a ratio of table values, for example $p_2/p_1 = (p/p^*)_{M_2}/(p/p^*)_{M_1}$. Figure 1 shows the ratios for $\gamma = 1.4$.

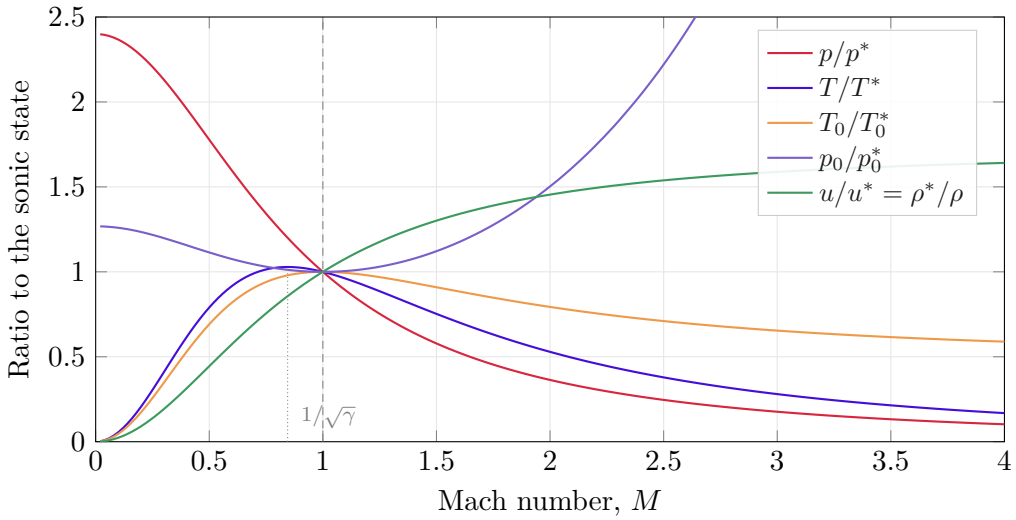


Figure 1: Rayleigh-flow ratios for $\gamma = 1.4$. All curves meet at 1 at $M = 1$. T/T^* peaks at $M = 1/\sqrt{\gamma}$; T_0/T_0^* peaks at $M = 1$.

2 How heating changes the flow, and thermal choking

2.1 The Rayleigh line

Plotted on a temperature–entropy diagram, the states one flow can reach form the *Rayleigh line* (Figure 2). Entropy is largest at $M = 1$. Since adding heat always raises entropy, heating drives a subsonic flow *up* toward $M = 1$ and a supersonic flow *down* toward $M = 1$; cooling does the opposite. The static temperature has its own maximum at

$$M = \frac{1}{\sqrt{\gamma}}, \qquad \frac{T_{\max}}{T^*} = \frac{(1+\gamma)^2}{4\gamma} \qquad (M = 0.845 \text{ and } T_{\max}/T^* = 1.029 \text{ for } \gamma = 1.4), \tag{7}$$

so on the subsonic branch between $M = 1/\sqrt{\gamma}$ and $M = 1$, adding heat *lowers* the static temperature: the energy added, and more, goes into accelerating the flow.

Table 1: Effect of heating ($q > 0$) on the flow. Cooling reverses every arrow.

Regime	M	u	p	ρ	T	T_0	p_0	s
Subsonic, $M < 1/\sqrt{\gamma}$	↑	↑	↓	↓	↑	↑	↓	↑
Subsonic, $1/\sqrt{\gamma} < M < 1$	↑	↑	↓	↓	↓	↑	↓	↑
Supersonic, $M > 1$	↓	↓	↑	↑	↑	↑	↓	↑

Heating always lowers the stagnation pressure (Table 1). This is why heat release in a combustor costs total pressure, and the loss grows with the Mach number at which heat is added.

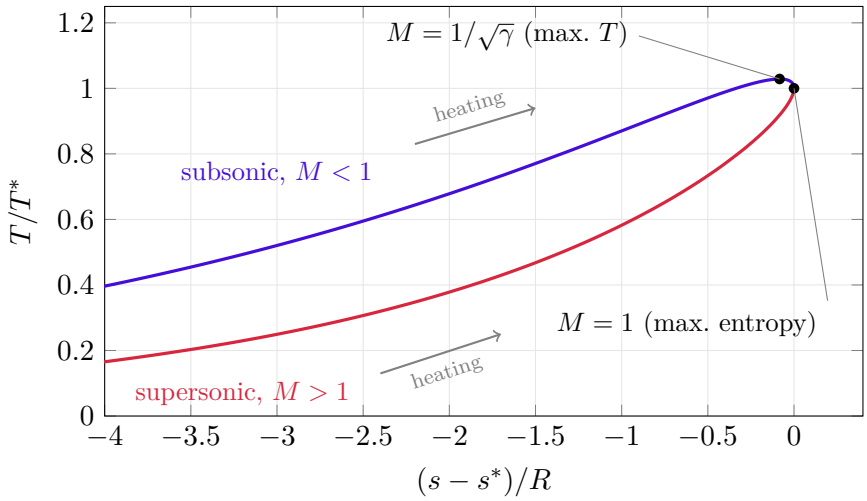


Figure 2: The Rayleigh line on a T–s diagram ($\gamma = 1.4$). Heating moves both branches toward the point of maximum entropy, $M = 1$.

2.2 Thermal choking

A flow can only be heated until it reaches $M = 1$, where $T_0 = T_0^*$. The largest heat input the inlet state allows is therefore

$$q^* = c_p\,(T_0^* - T_{01}) = c_p\,T_{01}\left[\frac{1}{(T_0/T_0^*)_{M_1}} - 1\right]. \tag{8}$$

If more heat than q^* is added, the given inlet state cannot be maintained: the flow is *thermally choked*. A subsonic flow responds by lowering its inlet Mach number, and so its mass flow, until it reaches $M = 1$ exactly at the exit. A supersonic flow adjusts through a shock wave in the duct, after which the now subsonic flow chokes at the exit.

3 What the two Rayleigh tabs compute

3.1 Rayleigh flow (ratio tables)

Enter γ and one known ratio. The calculator finds the Mach number and then evaluates all of (4)–(6). Several ratios have two Mach numbers for the same value, so the branch must be chosen:

- T_0/T_0^*, p_0/p_0^* and $(s^* - s)/R$: one subsonic and one supersonic solution.
- T/T^*: one solution below $M = 1/\sqrt{\gamma}$ and one above it ("below/above $T_{\max}$").
- p/p^* and u/u^* are single-valued and are inverted in closed form:

$$M^2 = \frac{(1+\gamma)/(p/p^*) - 1}{\gamma}, \qquad M^2 = \frac{u/u^*}{1 + \gamma - \gamma\,(u/u^*)}.$$

Two-branch ratios are solved by bisection on the chosen branch. Each ratio has a physical range, and values outside it are rejected (Table 2).

Table 2: Ranges of the Rayleigh ratios. Values in brackets are for $\gamma = 1.4$.

Ratio	Subsonic branch	Supersonic branch
T_0/T_0^*	$0 \to 1$	$1 \to (\gamma^2-1)/\gamma^2$ [0.4898]
p_0/p_0^*	$(1+\gamma)\left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)}$ [1.2679] $\to 1$	$1 \to \infty$
p/p^*	$1+\gamma$ [2.4] $\to 1$	$1 \to 0$
u/u^*	$0 \to 1$	$1 \to (1+\gamma)/\gamma$ [1.7143]
T/T^*	$0 \to (1+\gamma)^2/(4\gamma)$ [1.0286] at $M = 1/\sqrt{\gamma}$, then $\to 0$ as $M \to \infty$	

3.2 Rayleigh flow with heat addition

For a given inlet state and heat input, the calculator proceeds as follows.

1. Inlet stagnation state: $T_{01} = T_1\left(1 + \frac{\gamma-1}{2}M_1^2\right)$ and $p_{01} = p_1\,(T_{01}/T_1)^{\gamma/(\gamma-1)}$, with $c_p = \gamma R/(\gamma - 1)$.
2. Choking stagnation temperature $T_0^* = T_{01}/(T_0/T_0^*)_{M_1}$ and choking heat q^* from (8).
3. Exit stagnation temperature $T_{02} = T_{01} + q/c_p$, or as entered.
4. If $T_{02} > T_0^*$ the flow is choked (Section 2.2) and the sonic state is reported. Otherwise M_2 is found from T_{02}/T_0^* on the same branch as the inlet.
5. Exit state from the ratio tables: $p_2 = p_1\,(p/p^*)_{M_2}/(p/p^*)_{M_1}$, and likewise for T and p_0; $\rho_2 = p_2/(RT_2)$ and $u_2 = \rho_1 u_1/\rho_2$, since the mass flux is constant. The entropy change is $\Delta s/R = (s^* - s)_1/R - (s^* - s)_2/R$.

Example. Air ($\gamma = 1.4$, $R = 287.05$ J/(kg K)) enters at $M_1 = 0.3$, $T_1 = 300$ K, $p_1 = 100$ kPa. Then $T_{01} = 305.4$ K, $(T_0/T_0^*)_{M_1} = 0.3469$, $T_0^* = 880.5$ K and $q^* = 578$ kJ/kg. Adding $q = 300$ kJ/kg gives $T_{02} = 604.0$ K, $T_{02}/T_0^* = 0.686$ and $M_2 = 0.496$, with $p_2 = 83.7$ kPa.

4 Forced convection in a pipe

When a fluid flows through a pipe whose wall is hotter or colder than the fluid, heat is carried between the wall and the fluid by convection. The rate is described by Newton's law of cooling with a heat transfer coefficient h:

$$q'' = h\,(T_s - T_m), \tag{9}$$

where q'' is the wall heat flux, T_s the wall (surface) temperature and T_m the mean (bulk) temperature of the fluid, the temperature its mass flow carries. The coefficient h is not a property of the fluid; it depends on the flow, and is found from the dimensionless Nusselt number,

$$\mathrm{Nu} = \frac{hD}{k}, \qquad \mathrm{Re} = \frac{\rho V D}{\mu}, \qquad \mathrm{Pr} = \frac{\mu c_p}{k} = \frac{\nu}{\alpha}. \tag{10}$$

The Reynolds number decides whether the flow is laminar or turbulent. The Prandtl number compares how fast momentum and heat diffuse: about 0.7 for air, 7 for water at 20 °C and hundreds for oils.

4.1 Entrance regions and fully developed flow

Near the pipe inlet the velocity and temperature profiles are still developing and h is higher. After the hydrodynamic and thermal entrance lengths the profiles stop changing shape and Nu becomes constant (*fully developed*):

$$L_h \approx 0.05\,\mathrm{Re}\,D, \quad L_t \approx 0.05\,\mathrm{Re}\,\mathrm{Pr}\,D \quad \text{(laminar)}, \qquad L_h \approx L_t \approx 10D \quad \text{(turbulent)}. \tag{11}$$

The calculator uses fully developed values. For laminar flow of oils (large Pr) the thermal entrance length can exceed the pipe length; the average h is then higher than reported.

5 Nusselt number correlations

5.1 Laminar flow, $\mathrm{Re} < 2300$, fully developed

For fully developed laminar flow in a circular pipe the energy equation can be solved exactly. The Nusselt number is a constant that depends only on the wall condition:

$$\mathrm{Nu} = 3.66 \quad \text{(uniform wall temperature)}, \qquad \mathrm{Nu} = \tfrac{48}{11} = 4.36 \quad \text{(uniform wall heat flux)}. \tag{12}$$

In laminar flow the wall roughness has no effect on heat transfer or friction.

5.2 Turbulent flow: Gnielinski correlation (used by the calculator)

$$\mathrm{Nu} = \frac{(f/8)\,(\mathrm{Re} - 1000)\,\mathrm{Pr}}{1 + 12.7\sqrt{f/8}\,\left(\mathrm{Pr}^{2/3} - 1\right)}, \qquad 3000 < \mathrm{Re} < 5\times 10^6, \quad 0.5 < \mathrm{Pr} < 2000. \tag{13}$$

It is accurate to about 10% and rests on the analogy between momentum and heat transfer: the friction factor f measures how strongly turbulence near the wall transports momentum, and the same eddies transport heat. The calculator takes the Darcy friction factor from the Colebrook equation for the pipe's relative roughness ε/D,

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right), \tag{14}$$

the same as the Moody chart in the Frictional Flows calculator, so a rough pipe has both a higher pressure drop and a higher h. Roughness raises heat transfer less than it raises friction, so for very rough pipes this use of the analogy is an estimate.

5.3 Turbulent flow: Dittus–Boelter correlation (for comparison)

$$\mathrm{Nu} = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{n}, \qquad n = 0.4 \text{ (fluid heated)}, \quad n = 0.3 \text{ (fluid cooled)}, \tag{15}$$

for smooth pipes with $\mathrm{Re} > 10^4$ and $0.6 < \mathrm{Pr} < 160$. It is simple but can be in error by 25%. The exponent n accounts for how the temperature-dependent viscosity thins or thickens the fluid layer next to the wall.

5.4 Transitional flow, $2300 < \mathrm{Re} < 4000$

The flow may be laminar, turbulent, or switch between the two, and h is uncertain. The calculator reports the Gnielinski value with a warning; for design, check both limits. Once Nu is known, $h = \mathrm{Nu}\,k/D$. Figure 3 compares the correlations.

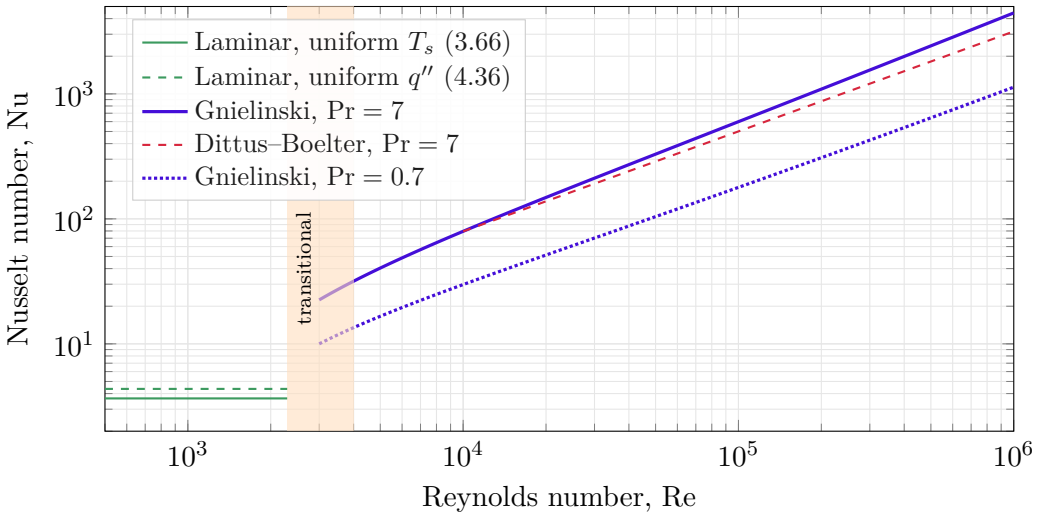


Figure 3: Fully developed Nusselt numbers in a smooth pipe. For this plot f is the smooth-pipe Petukhov value $f = (0.790 \ln \mathrm{Re} - 1.64)^{-2}$, which agrees with Colebrook for $\varepsilon = 0$ to within about 2%.

6 Energy balance along a heated pipe

With a pipe length L, the calculator also follows the fluid temperature. The heated surface is $A_s = \pi D L$, the mass flow $\dot{m} = \rho V \pi D^2/4$, and the heat gained by the fluid is

$$\dot{Q} = \dot{m}\, c_p\, (T_e - T_i). \tag{16}$$

6.1 Uniform wall temperature T_s

An energy balance on a slice of pipe of length dx, $\dot{m}\, c_p\, dT_m = h\,(T_s - T_m)\,\pi D\, dx$, integrates to an exponential approach of the fluid temperature to the wall temperature:

$$\frac{T_s - T_e}{T_s - T_i} = \exp\left(-\frac{h A_s}{\dot{m} c_p}\right) = e^{-\mathrm{NTU}}, \qquad \dot{Q} = h A_s\, \Delta T_{\mathrm{lm}}, \qquad \Delta T_{\mathrm{lm}} = \frac{T_e - T_i}{\ln\left[(T_s - T_i)/(T_s - T_e)\right]}. \tag{17}$$

The number of transfer units $\mathrm{NTU} = h A_s/(\dot{m} c_p)$ measures how close the outlet gets to the wall temperature: at $\mathrm{NTU} = 5$ the fluid has covered 99% of the difference. The log mean temperature difference is the correct average driving temperature difference; the arithmetic mean overestimates it.

6.2 Uniform wall heat flux q''

The heat input is fixed, so the fluid temperature rises linearly along the pipe and the wall stays a constant q''/h above it:

$$\dot{Q} = q'' A_s, \qquad T_e = T_i + \frac{\dot{Q}}{\dot{m} c_p}, \qquad T_s(x) = T_m(x) + \frac{q''}{h}. \tag{18}$$

The highest wall temperature is at the outlet, which is usually the design limit (for example, to avoid boiling or material damage).

Example. Water at 20 °C ($\rho = 998.2$ kg/m^3, $\mu = 1.002 \times 10^{-3}$ Pa s, $k = 0.598$ W/(m K), $c_p = 4182$ J/(kg K)) flows at 1 m/s in a copper pipe with $D = 25$ mm. Then Re = 24 900, Pr = 7.0, $f = 0.0247$, and Gnielinski gives Nu = 180 and $h = 4300$ W/(m^2 K); Dittus–Boelter gives Nu = 165.

7 Assumptions and limits

Rayleigh flow

Perfect gas with constant γ. At high temperatures (above about 1500 K for air) γ falls and dissociation begins, so the results become estimates. No friction: in long ducts friction and heating act together, and both push the flow toward $M = 1$.

Pipe convection

Circular pipe, fully developed flow, constant fluid properties. Evaluate the properties at the bulk mean temperature $(T_i + T_e)/2$; if it differs much from the values used, update them and recalculate. For large wall-to-fluid temperature differences, viscosity changes near the wall make the correlations less accurate.

Non-circular ducts

Turbulent correlations can be used with the hydraulic diameter $D_h = 4A/P$ (area over wetted perimeter). Laminar Nusselt numbers depend on the duct shape and differ from the circular-pipe values.

Accuracy

The Rayleigh relations are exact within their assumptions. The turbulent Nusselt correlations are empirical: about ±10% for Gnielinski and ±25% for Dittus–Boelter.

8 Notation

Symbol	Meaning	Units
M, a	Mach number; speed of sound $a = \sqrt{\gamma RT}$	–, m/s
p, T, ρ, u	static pressure, temperature, density, velocity	Pa, K, kg/m^3, m/s
p_0, T_0	stagnation (total) pressure and temperature	Pa, K
*	sonic (choking) reference state of the Rayleigh line	
γ, R, c_p	ratio of specific heats, gas constant, specific heat at constant pressure	–, J/(kg K)
q, q^*	heat added per unit mass; heat that chokes the flow	J/kg
s	specific entropy	J/(kg K)
h, q''	heat transfer coefficient; wall heat flux	W/(m^2 K), W/m^2
k, μ, ν, α	thermal conductivity; dynamic and kinematic viscosity; thermal diffusivity	W/(m K), Pa s, m^2/s
Nu, Re, Pr	Nusselt, Reynolds and Prandtl numbers	–
f, ε	Darcy friction factor; wall roughness	–, m
T_s, T_m, T_i, T_e	wall, bulk mean, inlet and exit fluid temperatures	K or °C
$\dot{m}$, $\dot{Q}$, A_s	mass flow rate, heat transfer rate, heated surface area	kg/s, W, m^2
NTU, ΔT_{lm}	number of transfer units; log mean temperature difference	–, K

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